Evolutionary Optimization of an Inexpensive Permanent Magnet Synchronous Wheel Motor

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Abstract - The objective of this project is to automate the design and optimization of an electrical machine through use of an Open Source software platform. A basic Permanent Magnet Synchronous Motor model called “LRK” popularly found in airplane modeling groups was scaled up and adapted for full-size automobile wheel-motor use, and a Finite Element Modeling and Intelligent Algorithm approach was taken to optimize the quality of the design.

I. INTRODUCTION

The ambitious goals of the project to implement a software framework both freely accessible and technically capable led the team to select the Open Source programs “Finite Element Method Magnetics” (FEMM) and “GNU Octave” (Octave) as the workhorses for a flexible and extensible design and optimization platform for electromechanical machines. The Finite Element (FE) pre- and post-processing is mainly done in Octave, and the FE processing is done by FEMM.

The application in view (automotive wheel motors) requires a machine with both high energy efficiency and power density, as well as the customary requirement of low cost, especially because it must be deployed in pairs.

A. The Electric Machine

The machine selected for a first attempt was the “Lukas Retzbach-Kulfuss brushless motor” (LRK) [1] and is depicted in Figures 1a/b. This Permanent Magnet Synchronous Motor (PMSM) with Radial Magnetic Flux (RMF) was chosen for its relative simplicity of construction, very low cogging torque, high power density, and potentially high energy efficiency and low cost.

Its concentrated windings make it easy to build, especially because they are only wound in every other tooth, yielding a stator set with only one copper winding per slot.

The favouring of standard-sized and standard-shaped “Nd-Fe-B” (Neodymium) PMs freely available in on-line shops over custom-designed wedge magnets is another important aspect of cost reduction.

B. The Software Platform

FEMM [3] is a sufficiently complete FE design and simulation tool. It is limited to two-dimensional design, either in a Cartesian plane or an Axisymmetric plane, with defined homogeneous Z depth. It can be automated via the internal language interpreter LUA 4.0, or externally via its interfaces to “Wolfram's Mathematica”, “MathWorks Matlab”, and “GNU Octave”.

GNU Octave [4] is a Free Software replacement for Matlab and achieves most of the basic functionality of numeric processing and data plotting. Its freedom, compatibility towards Matlab, and portability over the most popular computer operating systems make it a tool of choice in the academic medium. The fact that it costs nothing also helps.

This project's Octave scripts are held in a public repository [5] started and maintained by author Névoa and available for community development.

C. Structure of the Article

In chapter II we present the FE modelling of the chosen machine. Chapter III describes the optimization algorithms and their connection with the FE processor. Chapter IV describes the FE post-processing implemented by the project for the goal of optimization. Chapter V presents the obtained results. Chapter VI summarizes the conclusions and future work.

I. Finite Element Modelling

A. Two-dimensional modelling

The design plane chosen for representation of the electric machine in FEMM was the Cartesian plane that crosses the motor perpendicular to its rotation axis. This is the only choice of plane that usefully represents the radial magnetic flux and field variations necessary to the determination of torque. Because of this choice, and the fact that FEMM is a 2.5 dimensions program (it allows specification of the axis' “Z” length), the machine's ends are not represented at all.

As a consequence, any magnetic effects such as flux contributions and power losses that occur at those ends [6] are not taken into consideration in the model. This limitation, however, is currently viewed as secondary and is postponed to a later phase of the project. It is possible, though, that in certain cases where the width of the
windings is comparable to their depth, that the computed torque is significantly underestimated. This acts as an advantage to the design process, since the final output torque and power always result greater than the estimated amounts.

B. Mesh and Boundaries

As the machine is fully represented within two concentric circles, it is simple to define the boundary conditions of the exterior limits. They were set to a Dirichlet value of zero field strength. This is a realistic approach because the machine's exterior materials have a magnetic permeability much greater than air, which results in an effective magnetic "shield". The boundaries themselves are set outside of a reasonable layer of air surrounding the machine's exterior walls to detect any dispersion effects.

Considerable attention was paid to the sizing of the finite element mesh size. This being an automation project, the mesh sizes were defined as a fraction of the length of the component they reside in. Table I summarizes the mesh sizes chosen for each component class of the machine.

| Table I
<p>| Finite Element Mesh Sizes Picked for the Machine Model |</p>
<table>
<thead>
<tr>
<th>Machine Part</th>
<th>Finite Element Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exterior Structural Materials</td>
<td>Height/2</td>
</tr>
<tr>
<td>Air Gap &amp; Inter-pole Fillings</td>
<td>Height/3</td>
</tr>
<tr>
<td>PMs &amp; Rotor Cores</td>
<td>Height/4</td>
</tr>
<tr>
<td>Stator Cores &amp; Coils</td>
<td>Height/5</td>
</tr>
</tbody>
</table>

These mesh sizes were not validated under any process other than empirical experience with FEMM. To avoid large accuracy errors derived from wrong mesh sizes possibly set in the model, one could perform a comparison between the magnetic co-energy derivative and Maxwell's stress tensor to determine the error in torque computation and adjust the mesh until both techniques yield the same approximate result. This, however, is another sophisticated aspect better left for a later phase of the project.

C. Materials data base

All the materials' properties were defined in a file base for automated usage. The base includes the characteristics of structural materials such as air, stainless steel (SS316, SS416) and epoxy resin, magnetic materials such as one soft ferromagnetic Silicon-Iron alloy for the cores and 47 different types of Neodymium magnets, and one conductor material (copper). All materials are described by their electrical resistivity and magnetic permeability. The soft magnetic material is described by a dynamic permeability curve, whereas all others have a fixed permeability value. The list of 47 magnets was compiled from the Web offer of other than empirical experience with FEMM. To avoid large accuracy errors derived from wrong mesh sizes possibly set in the model, one could perform a comparison between the magnetic co-energy derivative and Maxwell's stress tensor to determine the error in torque computation and adjust the mesh until both techniques yield the same approximate result. This, however, is another sophisticated aspect better left for a later phase of the project.

D. Problem reduction

An attempt was made at reducing the machine problem to its minimum representative slice, in order to greatly reduce the computation load. This would entail modelling of a single set of phase coils, which would reduce the original machine to one half. Neumann boundary conditions would then be set at the newly created frontiers. Unfortunately, this approach revealed a complexity that was incompatible with the project's schedule, and was also left for a later phase. It is not a simple task to automate the subdivision of a variable geometry model that is capable of rotor-stator relative displacement.

E. Physical coherence

A large set of ground rules for geometric coherence was defined into the model scripts. It ensures that the machine is drawn in FEMM with realistic care, such as not having any overlapping parts. It also implements a few design policies chosen from the authors' scarce experience in machine design, such as the stator slots being dimensioned to exactly allow the cross-section area of copper dictated by the conductor diameter and number of winding turns, leaving no air between stator poles. The chosen copper conductor diameter dictates the maximum current allowed for the machine, which has passive air cooling.

F. Base model

The basic starting point machine was the model aeroplane "LRK" motor, which was then adapted for use as a full-scale automotive wheel motor and realistically modelled. The main differences between the source LRK machine and this project's reference machine are the size and rated output power (400W the former, 20kW the latter) and the stator geometry near the axis of the machine (this project's machine has a large hole in the centre to allow the presence of a hydraulic brake drum).

Figure 2 presents ¼ of the reference machine. The stator has 12 poles, 6 of them actively wound, and the rotor has 14 poles.

Fig. 2 – Base reference model (up-scaled LRK), with enlarged detail of a rotor pole PM. Notice the high density meshing inside the interstitial glue regions surrounding each magnet. Only Phase B circuit is completely visible (2nd stator tooth, counting clockwise).

The rotor poles are constructed from multiple rectangular section magnets arranged into curved poles separated by air. The magnets are glued to the rotor core with epoxy
adhesive. This strategy allows reuse of inexpensive standard shaped magnets.

The stator coils have 20 turns and are dimensioned for 50A currents. It is a three phase machine, and the simulated current wave shape is trapezoidal.

G. Model simplification

To reduce the FE computational load, the model had to be simplified without loss of representation. Thus the highly detailed PM arrangements in the rotor where replaced by single equivalent-size curved magnets without interstitial glue (see Fig.3). This replacement is however done while respecting the original dimensions of the selected data base magnet, allowing machine construction with it.

Another simplification was done in the air gap region that sides the rotor poles (see Fig.4). This region does not contribute to torque production as greatly as the region between rotor and stator and does not need such a fine mesh. A division was drawn between these two air gap regions and the mesh defined accordingly. This enhancement also allows us to select different materials for rotor inter-pole filling as will be seen in the next section in parameter 2.

As a result of these simplifications, the computation time for the base model was reduced by 78.8% (from 72,922 nodes down to 15,435), without perceivable impact on torque output.

H. Model parameters

The automation of the design implies a set of model parameters which can be manipulated to create different “candidate” machines. Table II describes the variable input parameters used to explore the machine’s possibilities. In addition, a fixed set of parameters was defined according to the application’s requirements: the maximum outer radius, the minimum inner radius, and the maximum axial depth. The intention is to find the best possible motor design that fits into a specific wheel on a case-by-case approach.

<table>
<thead>
<tr>
<th>#</th>
<th>Parameter</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Magnet Type</td>
<td>1 .. 47, integer</td>
</tr>
<tr>
<td>2</td>
<td>Inter-pole filling</td>
<td>0.2 = {Air, Iron, Halbach}</td>
</tr>
<tr>
<td>3</td>
<td>Stator Material</td>
<td>1 = {Iron, Epoxy}</td>
</tr>
<tr>
<td>4</td>
<td>Air Gap thickness</td>
<td>0.5 .. 3, step=0.05mm</td>
</tr>
<tr>
<td>5</td>
<td>Pole Multiplier</td>
<td>1 .. 10, integer</td>
</tr>
<tr>
<td>6</td>
<td>Rotor Core Height</td>
<td>3 .. 30, step=0.5mm</td>
</tr>
<tr>
<td>7</td>
<td>Stator Core Height</td>
<td>3 .. 30, step=0.5mm</td>
</tr>
<tr>
<td>8</td>
<td>Rotor Occupation Factor</td>
<td>0.05 .. 0.95, step=0.10</td>
</tr>
<tr>
<td>9</td>
<td>Maximum Current</td>
<td>25 .. 85, step=5A</td>
</tr>
<tr>
<td>10</td>
<td>Pole Coil Turns</td>
<td>5 .. 200, step=5</td>
</tr>
</tbody>
</table>

Within this parameter scheme the basic machine model used as comparison reference had the following parameters: [47, 0, 3, 1, 8, 8, 0.5, 50, 20].

I. Optimization Algorithms

A. Search space

The full combinatorial search space covers around 2x10^{12} possibilities; fortunately most of these are not geometrically possible, and the real search space is reduced to an estimated 3.6x10^9 valid candidates. This search space was divided into 3 sections according to a fundamental question: which inter-pole filling material is best? So 3 optimization runs where done in separate, one for each value of parameter 2.

To find our optimized versions of the machine, we built a simple intelligent system that combines an Evolutionary or Genetic Algorithm (GA) with a Hill Climber (HC) optimization algorithm.

Both of these algorithms manipulate parameter sets in a controlled trial-and-error fashion, but in different approaches and with different results. The first one is very good at finding the best candidate “families” of parameter sets (global optima neighbourhoods), and the second one is good at fine-tuning a parameter set to its maximum score (local optima). Utilizing the GA to find the absolute optimal solution has proven too heavy a job for any computation platform; the final convergence takes too long, thus the hand-over to the HC produces faster results. This hand-over is a simple job and has been performed manually while observing the convergence speed of the optimization system, but is scheduled to be fully automated when time permits.

B. Genetic Algorithm

Reference [7] presents the essence and workings of the GA algorithm. Our implementation in Octave reuses the code built for a previous project [8] of author Veiga, which also required the maximization of an aptitude value. For this application the following functions had to be redefined:

1. Validation: geometric validation of a parameter set before feeding it to the FE processor;
2. **Mutation**: random change in one parameter of the candidate's set, while observing the stipulated parameter limits and set validity;

3. **Crossover**: random recombination of two candidates' parameters into two new candidates, while observing validity of the new sets;

4. **Aptitude**: the “score” or “merit” of each candidate set; this project values energy efficiency more than output power, so the elected aptitude value to maximize is

\[
\text{score} = \text{torque} \times \text{efficiency}^2.
\]  

(1)

The efficiency value is in percentage (0-100%). The torque and efficiency values are the mean values of the simulation samples taken from the FE processor.

A. **Hill Climber**

Reference [9] presents the essence and workings of the HC algorithm, whose implementation was also reused from [8].

I. **PROCESSING AND POST PROCESSING**

A. **Torque computation**

For FE post-processor extraction of the machine's output torque, the Maxwell Stress Tensor [10] available in FEMM was used. The region selected for this 2-D integral is the entire rotor set, comprising the PMs as well as the iron core and remaining structural materials behind it.

FEMM only allows the simulation of a single sinusoidal frequency of fields at a time. This is neither compatible with the trapezoidal current wave form used with our machine, nor with the presence of PMs (0Hz fields). To compute the full effects of a trapezoidal wave via sinusoidal waves, a Fourier decomposition into harmonic frequencies and therefore many more simulations would be necessary. And to include the static fields of the rotor magnets in the simulation some equivalent sinusoidal field source would be required.

These methods were discarded in favour of a more cost-efficient one: sequential static simulation (0 Hz fields). In this context, the PMs' field is observed by the FE processor and the electric currents are manipulated in instantaneous values instead of phasor vectors. As a consequence, the FE post-processor has no way of calculating induced currents or hysteresis losses. This limitation is well known but is better left for a later phase. The author of FEMM has agreed to develop an extension of the program's API to allow access to the matrix of finite elements inside the model, which will permit a Fast Fourier Transform analysis to externally determine these losses from our 0Hz simulation.

B. **Efficiency computation**

For efficiency extraction, an analytic approximation was necessary. The expression used is

\[
\text{efficiency} = 100 \times \frac{\text{output } P}{\text{input } P}
\]  

(2)

which uses the output power

\[
\text{output } P = \text{torque} \times \omega
\]  

(3)

and input power

\[
\text{input } P = \text{output } P + \text{copper losses}
\]  

(4)

\(\omega\) is the angular speed of the rotor, in rad/s. The copper losses are obtained via

\[
copper P = \sum (\text{voltage} \times \text{current})
\]  

(5)

for all 3 phases. The voltage for each phase is obtained by direct extraction from FEMM, which finds the resistive drop for each phase circuit. These values do not include Electro Motive Force (EMF) because these are zero-frequency simulations. EMF values are nonetheless externally computed by differentiating the phase linked flux (also provided by FEMM) over the time steps, in order to find the approximate phase terminal voltage

\[
\text{phase voltage} = \text{resistive drop} + \text{EMF}
\]  

(6)

A. **Outputs for Optimization**

For optimization purposes, only 3 instants are simulated, for 3 different rotor positions: start, middle, and end of a commutation step. The commutation “steps” here referred are the 6 divisions of a full electric period of a 3-phase trapezoidal current wave. The mean values of torque and efficiency are used to compute the score.

\[ \text{Figure 5 – Post-processing outputs for candidate model evaluation: only 3 samples taken for torque and efficiency over one full rotation step.} \]

B. **Outputs for Confirmation**

Once a good candidate model is found, a full rendering of its steady behaviour at full speed and full torque is plotted. First, an automated study (a sequence of 1 step * 3 instants runs) is done to find the best commutation advance angle that maximizes the mean torque; then a (3 steps * 20 instants) simulation run is used to plot detailed torque, power, efficiency, voltage, EMF, and current graphs.

\[ \text{Figure 6 – Post-processing outputs for optimized model evaluation: 15 samples taken for torque and efficiency over 6 rotation steps (one full electric period).} \]

This last run allows detailed observation of the motor physics as well as confirmation of the correct implementation of the model's current switching and mechanical synchronism algorithms.
I. RESULTS

The optimization results produced by the framework in the three categories defined by parameter 2 (inter-pole filling) are now presented.

A mini-cluster of 7 Personal Computers (PCs) was used for this research project (off-hours CPU time kindly provided by “Laboratório Militar de Produtos Químicos e Farmacêuticos”, Lisbon, Portugal), but we would gladly have used 70 or even 700 PCs, should they be available.

A. Optimized “Air” machine

The winner set with “Air” inter-pole filling (a.k.a. “salient pole rotor”) is described in Table III.

<table>
<thead>
<tr>
<th>#</th>
<th>Parameter</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Magnet Type</td>
<td>DxWxH=40x10x5; Hc=955kA/m</td>
</tr>
<tr>
<td>2</td>
<td>Inter-pole filling</td>
<td>Air</td>
</tr>
<tr>
<td>3</td>
<td>Stator Material</td>
<td>Iron</td>
</tr>
<tr>
<td>4</td>
<td>Air Gap thickness</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>5</td>
<td>Pole Multiplier</td>
<td>6 =&gt; 72 stator poles / 84 rot. p.</td>
</tr>
<tr>
<td>6</td>
<td>Core Height</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>7</td>
<td>Stator Core Height</td>
<td>7 mm</td>
</tr>
<tr>
<td>8</td>
<td>Rotor Occupation Factor</td>
<td>90%</td>
</tr>
<tr>
<td>9</td>
<td>Maximum Current</td>
<td>80 A</td>
</tr>
<tr>
<td>10</td>
<td>Pole Coil Turns</td>
<td>5 turns per tooth coil</td>
</tr>
</tbody>
</table>

The strategies identified by the framework for a high score in this model were clearly a reduction of the air gap thickness and an increase in pole count, current intensity, and pole width – all these values are fixed at their respective allowed limits.

The outputs of this model (presented in Figure 7) are:

- Torque = 198.8 Nm
- Efficiency = 90%

Figure 7 – Post-processing output for optimized “Air” model, over 3 electric switching steps (half an electric period).

A. Optimized “Iron” machine

For the “Iron” filled option (a.k.a. “smooth pole rotor”), the winner set is described in Table IV.

<table>
<thead>
<tr>
<th>#</th>
<th>Parameter</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Magnet Type</td>
<td>DxWxH=20x10x5; Hc=955kA/m</td>
</tr>
<tr>
<td>2</td>
<td>Inter-pole filling</td>
<td>Silicon-Iron alloy</td>
</tr>
<tr>
<td>3</td>
<td>Stator Material</td>
<td>Iron</td>
</tr>
<tr>
<td>4</td>
<td>Air Gap thickness</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>5</td>
<td>Pole Multiplier</td>
<td>6 =&gt; 72 stator poles / 84 rot. p.</td>
</tr>
</tbody>
</table>

Figure 8 – Post-processing output for optimized “Iron” model.

It is visible in Fig.8 a reduction in harmonic distortion of the torque wave, which suggests a beneficial preference of embedded rotor magnets over salient ones (to minimize iron losses from high-frequency harmonics).

A. Optimized “Halbach” machine

The “Halbach” filled rotor option is described in Table V.

<table>
<thead>
<tr>
<th>#</th>
<th>Parameter</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Magnet Type</td>
<td>DxWxH=3x3x3; Hc=955kA/m</td>
</tr>
<tr>
<td>2</td>
<td>Inter-pole filling</td>
<td>Halbach magnets</td>
</tr>
<tr>
<td>3</td>
<td>Stator Material</td>
<td>Iron</td>
</tr>
<tr>
<td>4</td>
<td>Air Gap thickness</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>5</td>
<td>Pole Multiplier</td>
<td>2 =&gt; 24 stator poles / 28 rot. p.</td>
</tr>
<tr>
<td>6</td>
<td>Core Height</td>
<td>5.5 mm</td>
</tr>
<tr>
<td>7</td>
<td>Stator Core Height</td>
<td>4.5 mm</td>
</tr>
<tr>
<td>8</td>
<td>Rotor Occupation Factor</td>
<td>70%</td>
</tr>
<tr>
<td>9</td>
<td>Maximum Current</td>
<td>70 A</td>
</tr>
<tr>
<td>10</td>
<td>Pole Coil Turns</td>
<td>15 turns per tooth coil</td>
</tr>
</tbody>
</table>

Again the framework converges to reduced air gap and increased current intensity, but not to increased pole count and pole width. This apparently stresses a need for a minimum length of Halbach magnet stacking along the inter-polar rotor region, dictating that this region cannot be too short; hence the relatively fewer and narrower poles.

The outputs are (from Figure 9):

- Torque = 150.1 Nm;
- Efficiency = 90.1%.

It is visible a higher harmonic content in the torque wave than in the other two options.
A. Comparison to original machine

Figures 10, 11, and 12 present the topologies of the 3 “winner” machine candidates in the optimization process.

Table VI summarizes the output results.

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameters</th>
<th>Torque (Nm)</th>
<th>Eff. (%)</th>
<th>Power (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>[17,0,0.5,6,3,7,0,9,80,5]</td>
<td>198.8</td>
<td>90.0</td>
<td>26.9</td>
</tr>
<tr>
<td>Iron</td>
<td>[11,1,0.5,6,3,7,0.9,80,5]</td>
<td>202.2</td>
<td>90.1</td>
<td>27.3</td>
</tr>
<tr>
<td>Halbach</td>
<td>[27,2,0.5,2,5,4,0.7,70,15]</td>
<td>150.1</td>
<td>90.1</td>
<td>20.3</td>
</tr>
</tbody>
</table>

I. Conclusions

A. Project outcome

The software framework is not yet satisfying but holds promise in automated optimization of customized wheel motors. It is capable of designing machines according to given requirements and optimizing them through a defined parameter set.

Our GA is computationally costly and requires further enhancement (only 17,500 candidates were computed in the time frame allowed). Given the extent of the search space, a study of parameter-output sensitivity is advised to rule out the ones with the least impact.

The Halbach arrangement of the PMs is not yet discarded in spite of discouraging results. Halbach is best applied in core-less machines, which is not the case. Also, our geometric constraints make it difficult to find and study this alternative.

Our results are merely indicative of the motor’s design quality, that currently ignores the losses in the different construction materials. This fact, coupled with the so far obtained maximum efficiency of 90%, spells great difficulties for this project in achieving circa 95% efficiency levels without considering different machine topologies.

B. Future work

Several improvements must be made to the framework before it can be used for reliable machine design:

1. Inclusion of induced current and hysteresis losses;
2. More realistic current wave simulation;
3. Validation of PM field limits for demagnetization;
4. Reliable model reduction;
5. Thermal simulation;
6. Mechanical simulation;
7. Three Dimensional modelling.

Inclusion of Artificial Neural Networks as a complement to the FE processor is also a possibility for a massive speed increase of the framework.

The inclusion of financial constraints (materials and manufacturing) into the optimization mechanism is quite feasible via the materials data base.

References